IE521 Convex Optimization

Introduction

Instructor: Niao He

Jan 18, 2017

About Me

ILLINOIS
UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN

- Assistant Professor,
  UIUC, 2016 –

- Ph.D. in Operations Research,
  M.S. in Computational Sci. & Eng.
  Georgia Tech, 2010 – 2015

- B.S. in Mathematics,
  University of Sci. & Tech. of China,
  2006 – 2010

About The Course

Convex Optimization

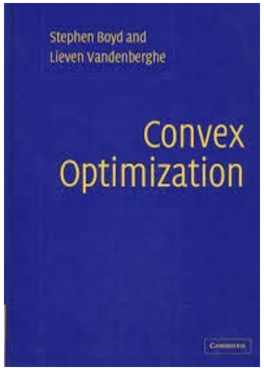
- **Emphasize** an in-depth understanding of basic convex analysis and disciplines: convex sets and functions, duality theory, optimality conditions.

- **Concentrates** on structural convex programming, e.g., linear, quadratic, semidefinite programs, and polynomial-time algorithms, e.g. Interior-point methods.

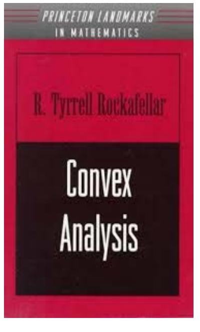
- **Enhance** the ability of recognizing and solving convex optimization problems that arise in applications.

Course Details

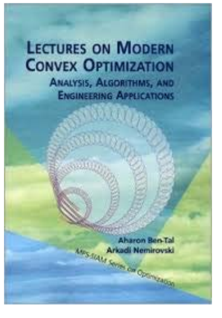
- **Prerequisites**: no formal ones, but assume knowledge in
  - linear algebra, multivariate calculus
  - mathematical thinking and modeling
- **Textbooks:** no required ones, but recommend to read the listed references
  - *Boyd & Vandenberghe. Convex Optimization. Cambridge University Press. (2004)*



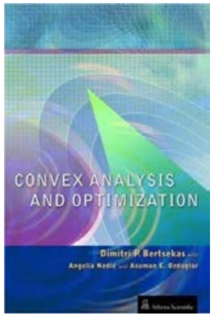
Boyd & Vandenberghe



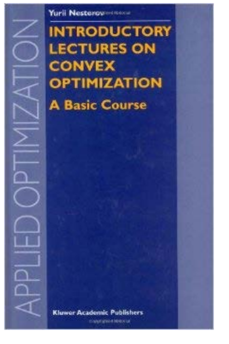
Rockafellar



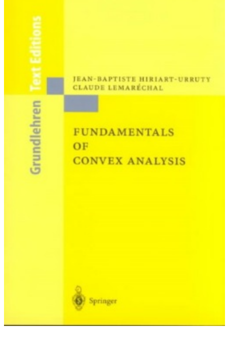
Nemirovski



Bertsekas, Nedich & Ozdaglar



Nesterov



Hiriart-Urruty & Lemaréchal

Course Details

- **Evaluation:**
  - Homework (30%): due bi-weekly
  - Midterm Exam (30%): exact date TBD
  - Final Exam (30%): exact data TBD
  - Class Participation (10%): attendance is required

- **Where to get help:**
  - Email: niaohe@illinois.edu with [IE 521] in your subject
  - Office Location: 211 Transport Building
  - Office Hours: Tue. 9:00-10:00 or by appointment via email
  - Teaching Assistant: TBD

Why you Should take the course

Mathematical Optimization

- General form of an optimization problem:

$$\min_{x=(x_1,\ldots,x_n)} \quad f_0(x)$$

$$\text{s.t.} \quad f_i(x) \leq 0, i = 1, \ldots, k$$

$$h_j(x) = 0, j = 1, \ldots, \ell$$

Objective function

Equality Constraints

Inequality Constraints

Decision variables

- Optimal solution x_*:
  - attains smallest objective value
  - Satisfies all constraints

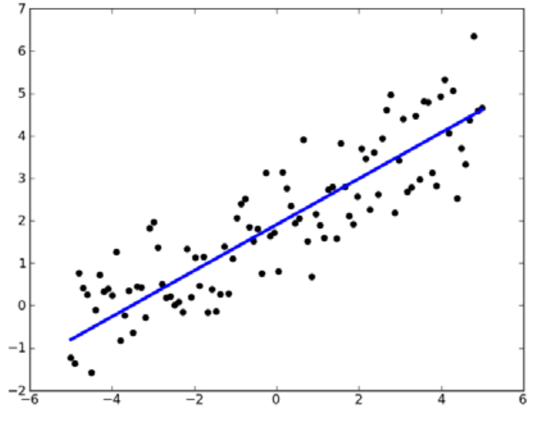
Optimization is everywhere.

- Computer Science
  - Machine learning, computer vision, etc.
- Electrical Engineering
  - Signal and image processing, control and robotics, etc.
- Finance
  - Portfolio selection, asset pricing and arbitrage, etc.
- Industrial Engineering
  - Supply chain, revenue management, transportation etc.
- Statistics
  - Parametric estimation, Bayesian inference, etc.

Examples

- Portfolio optimization
  - Variables: amounts invested in different assets
  - Constraints: budget, short sell
  - Objective: overall risk or total return

- Linear regression
  - Variables: coefficients of hyperplane
  - Constraints: regularity or sparsity
  - Objective: sum of squared residuals

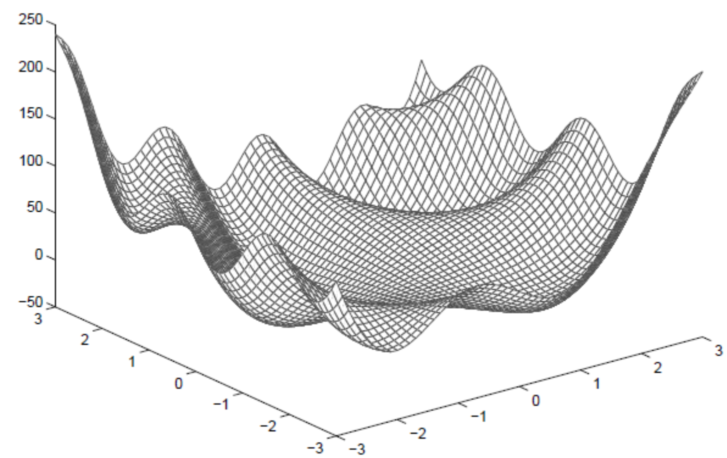


Solving Optimization

ILLINOIS
UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN

- Generally speaking, can be very difficult:
  - Many local minimum solutions of the objective
  - Constraints can be very complicated
  - Greedy search is often impossible

$$\begin{aligned} \min_x \quad & f_0(x) \\ \text{s.t.} \quad & f_i(x) \leq 0, i = 1, \ldots, k \\ & h_j(x) = 0, j = 1, \ldots, \ell \end{aligned}$$



When is an optimization problem easy to solve?

Easy or Hard?

$$\min_x \ c^T x \quad \text{s.t.} \ Ax \le b$$

$$\min_x \ P(x)$$
$$\text{where } P(x) \text{ is polynomial}$$

Linear Optimization

Polynomial Optimization

Easy

Hard

Easy or Hard?

$$\min_x \quad \frac{1}{2}x^TQx + b^Tx + c$$
where Q is positive semidefinite

Quadratic Optimization

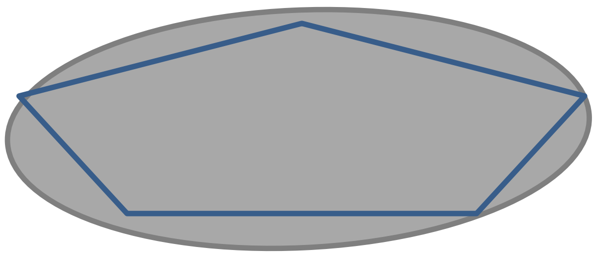
Easy

$$\min_x \quad \frac{1}{2}x^TQx + b^Tx + c$$
where Q is indefinite

Quadratic Optimization

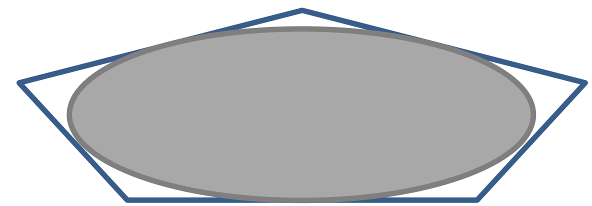
Hard

Easy or Hard?



Find minimum volume ellipsoid

Hard



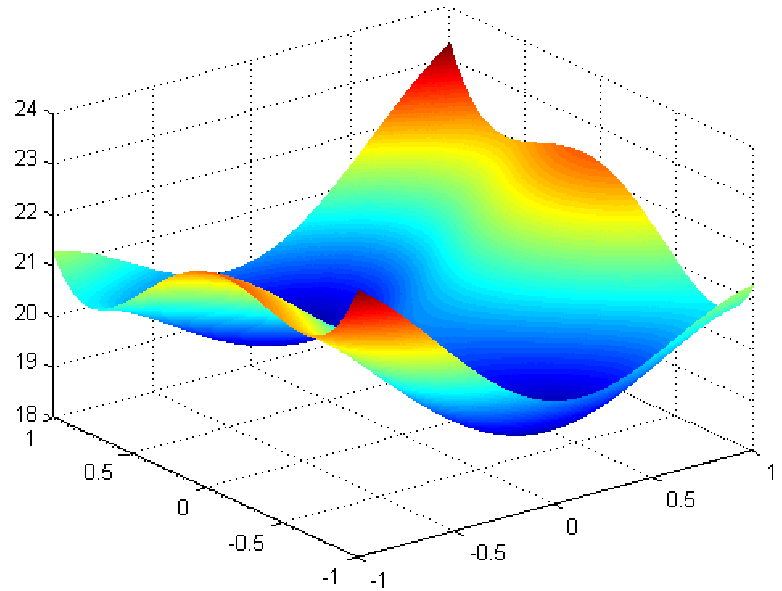
Find maximum volume ellipsoid

Easy

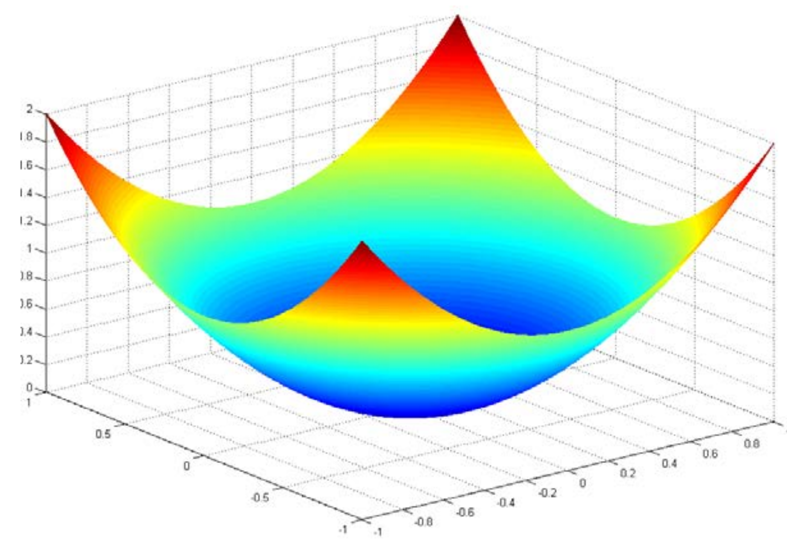
Example from L. Xiao, CS286 seminar

Why Convexity?

> “ The great watershed in optimization isn’t between linearity and nonlinearity, but convexity and nonconvexity.”
>
> — R. Rockafellar, SIAM Review 1993



Non-Convex Optimization



Convex Optimization

Course Goals

- How to characterize convexity?
- How to recognize whether a problem is convex?
- How to solve a convex problem?
- How to apply and implement the theory and algorithm to address real-world applications?

Course Topics

- Fundamental Theory of Convex Analysis
  - Convex set
  - Convex function
  - Convex program
  - Convex geometry
  - Constrained problems and Lagrangian duality
  - Optimality conditions

- Disciplined Convex Programming
  - Linear Programming
  - Conic Quadratic Programming
  - Semidefinite Programming

Course Topics (cont'd)

- Polynomial Solvability of Convex Optimization
  - Center-of-gravity
  - Ellipsoid method
  - Interior point method

- Algorithms for Constrained Convex Optimization
  - Subgradient method
  - Cutting plane method
  - Bundle method
  - Dual method

Course Topics (cont'd)

- Selective Application and Implementation
  - CVX tutorial
  - Robust optimization
  - Sparsity optimization
  - Statistical inference and machine learning
- Recent Advance in Large-Scale Convex Optimization
  - Accelerated gradient descent
  - Stochastic gradient descent
  - Several other algorithms if time permits

Why you Should Not take the course

Situation I

- If you are looking for general and intro-level optimization course, you should check this one:

ECE 490: Introduction to Optimization (this semester, by R. Srikant)

Situation II

- If you are looking for an advanced optimization course, you should check this one:

IE 598: Advanced Topics in Continuous Optimization (this semester, by R. Sun) or

IE 598: Big Data Optimization (spring 2018, by N. He)

Situation III

- If you are looking for specific optimization, e.g., linear optimization, or discrete optimization, you should check

IE 511: Integer Programming (this semester)

IE 411: Optimization of Large Systems (fall 2017)

IE 521 : Advanced Nonlinear Programming (spring 2018)

Otherwise

- See you next week.