IE521 Convex Optimization

Introduction

Instructor: Niao He

Jan 14, 2019
About Me

• Assistant Professor,  
  *UIUC, 2016 –*

• Ph.D. in Operations Research,  
  M.S. in Computational Sci. & Eng.  
  *Georgia Tech, 2010 – 2015*

• B.S. in Mathematics,  
  University of Sci. & Tech. of China,  
  2006 – 2010
Convex Optimization

- **Emphasize** an in-depth understanding of convex analysis and disciplines: convex geometry, duality theory, optimality conditions.

- **Concentrates** on structural convex programming, e.g., linear, quadratic, semidefinite programs, and polynomial-time algorithms, e.g. Interior-point methods.

- **Enhance** the ability of recognizing and solving convex optimization problems that arise in applications.
About You

- PhD or Master?
- ISE, ECE, CS?
- Took any optimization courses?
- Expectations of the course?

Please fill in the entry survey and return back to me.
Course Details

• **Prerequisites**: no formal ones, but assume knowledge in
  – linear algebra, multivariate calculus
  – mathematical thinking and modeling

• **Textbooks**: no required ones, but recommend to read the
  listed references
Course Admin

• Syllabus & Website
  [http://niaohe.isel.illinois.edu/IE521/](http://niaohe.isel.illinois.edu/IE521/)

• Where to get help:
  – Email: niaohe@illinois.edu with [IE 521] in your subject
  – Office Location: 211 Transportation Building and 106 CSL
  – Teaching Assistants: Juan Xu and Xiaobo Dong
  – Office Hours: TBD
Course Evaluation

• Grading Policy:
  – Homework (45%): due bi-weekly
  – Midterm Exam (20%): exact date TBD
  – Final Exam or Project (30%): exact date TBD
  – Class Participation (5%): attendance is required
  – Bonus (up to 5%): catching up typos and mistakes in lecture notes/homeworks/solutions
Why you Should take the course
Mathematical Optimization

• General form of an optimization problem:

\[
\min_{x=(x_1,\ldots,x_n)} f_0(x)
\]

s.t. \( f_i(x) \leq 0, i = 1, \ldots, k \)

\( h_j(x) = 0, j = 1, \ldots, \ell \)

• Optimal solution \( x_* \):
  – attains smallest objective value
  – Satisfies all constraints
Optimization is Everywhere.

• Computer Science
  – Machine learning, computer vision, etc.

• Electrical Engineering
  – Signal and image processing, control and robotics, etc.

• Finance
  – Portfolio selection, asset pricing and arbitrage, etc.

• Industrial Engineering
  – Supply chain, revenue management, transportation etc.

• Statistics
  – Parametric estimation, Bayesian inference, etc.
Examples

• Portfolio selection
  – Variables: amounts invested in different assets
  – Constraints: budget, short sell
  – Objective: overall risk or total return

• Linear regression
  – Variables: coefficients of hyperplane
  – Constraints: regularity or sparsity
  – Objective: sum of squared residuals
Solving Optimization

• Generally speaking, can be very difficult:
  – Many local minimum solutions of the objective
  – Constraints can be very complicated
  – Greedy search is often impossible

\[
\begin{align*}
\min_x & \quad f_0(x) \\
\text{s.t.} & \quad f_i(x) \leq 0, \ i = 1, \ldots, k \\
& \quad h_j(x) = 0, \ j = 1, \ldots, \ell
\end{align*}
\]

When is an optimization problem easy to solve?
Easy or Hard?

Linear Optimization

$$\min_{x} c^T x$$

s.t. $Ax \leq b$

Polynomial Optimization

$$\min_{x} P(x)$$

where $P(x)$ is polynomial

Easy

Hard
Easy or Hard?

Minimize: \[
\min_x \frac{1}{2} x^T Q x + b^T x + c
\]

where \(Q\) is positive semidefinite

Quadratic Optimization

Easy

Minimize: \[
\min_x \frac{1}{2} x^T Q x + b^T x + c
\]

where \(Q\) is indefinite

Quadratic Optimization

Hard
Easy or Hard?

Find minimum volume ellipsoid

Hard

Find maximum volume ellipsoid

Easy

Example from L. Xiao, CS286 seminar
Why Convexity?

“The great watershed in optimization isn’t between linearity and nonlinearity, but convexity and nonconvexity.”
— R. Rockafellar, SIAM Review 1993

Non-Convex Optimization

Convex Optimization
Course Goals

• How to characterize convexity?
• How to recognize whether a problem is convex?
• How to solve a convex problem?
• How to apply and implement the theory and algorithm to address real-world applications?
Course Topics

• **Fundamental Theory of Convex Analysis**
  – Convex set
  – Convex function
  – Convex program
  – Convex geometry
  – Constrained problems and Lagrangian duality
  – Optimality conditions

• **Disciplined Convex Programming**
  – Linear Programming
  – Conic Quadratic Programming
  – Semidefinite Programming
Course Topics (cont’d)

• Polynomial Solvability of Convex Optimization
  – Center-of-gravity
  – Ellipsoid method
  – Interior point method

• Algorithms for Constrained Convex Optimization
  – Subgradient method
  – Cutting plane method
  – Bundle method
  – Dual method
  – Primal-Dual method
Course Topics (cont’d)

• Selective Application and Implementation
  – CVX tutorial
  – Robust optimization
  – Statistical inference
  – Machine learning
  – Reinforcement learning
  – Others
Why you may **Not** take the course
Situation I

• If you are looking for an intro-level optimization course, you should check this one:

ECE 490: Introduction to Optimization (this semester)
Situation II

- If you are looking for specific optimization, you should check

  IE 511: Integer Programming (this semester)
  IE 411: Optimization of Large Systems (this semester)
  CS 544: Optimization in Computer Vision (this semester)
  ECE 580: Optimization by Vector Space Methods (this semester)
  IE 598: Optimization for Deep Learning (this semester)
Situation III

• If you are looking for an advanced optimization course, you should check this one:

IE 521: Advanced Nonlinear Programming (spring 2020)
IE 598: Big Data Optimization (spring 2020)
Otherwise

• See you next Wednesday.